

$\Omega\Omega$ interaction from 2+1 flavor lattice QCD

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Abstract

We investigate the interaction between Ω baryons in the 1S_0 channel from 2+1 flavor lattice QCD simulations. On the basis of the HAL QCD method, the $\Omega\Omega$ potential is extracted from the Nambu-Bethe-Salpeter wave function calculated on the lattice by using the PACS-CS gauge configurations with the lattice spacing $a \simeq 0.09$ fm, the lattice volume $L \simeq 2.9$ fm and the quark masses corresponding to $m_\pi \simeq 700$ MeV and $m_\Omega \simeq 1970$ MeV. The $\Omega\Omega$ potential has a repulsive core at short distance and an attractive well at intermediate distance. Accordingly, the phase shift obtained from the potential shows moderate attraction at low energies. Our data indicate that the $\Omega\Omega$ system with the present quark masses may appear close to the unitary limit where the scattering length diverges.

Introduction Strange dibaryons have been attracting considerable interests both theoretically and experimentally in hadron physics. In particular, the H -dibaryon with (strangeness) $=-2$ [1] and the $N\Omega$ dibaryon with (strangeness) $=-3$ [2] are considered to be the promising dibaryon states due to the absence of Pauli repulsions among valence quarks at short distance (see the reviews, [3, 4]). In recent years, the numerical and theoretical progresses in lattice gauge theories made it possible to attack such a problem directly from quantum chromodynamics (QCD) (See e.g.[5, 6, 7] and references therein.)

The purpose of this letter is to extend our previous works on the (strangeness) $=-2$ systems such as H -dibaryon [5] and $N\Xi$ [8] and the (strangeness) $=-3$ system as $N\Omega$ [9] to the (strangeness) $=-6$ $\Omega\Omega$ system in 2+1 flavor lattice QCD. In our approach (the HAL QCD method), the baryon-baryon potential is extracted from the Nambu-Bethe-Salpeter (NBS) wave function calculated on the lattice: Such a potential deduced in lattice QCD is guaranteed to reproduce physical observables (e.g. the scattering phase shift) by construction [10]. The HAL QCD method has several advantages over the conventional finite volume method [11]: (i) The finite volume effect is highly suppressed due to the short range character of baryon potentials, (ii) the ground state saturation of the two-particle system is not required for extracting the potential, since the same potential distates all the scattering states on the lattice, and (iii) physics behind the baryon-baryon interaction can be easily grasped from the spatial structure of the potential. Further details on these points are discussed in [12, 13].

There exist several investigations so far on the $\Omega\Omega$ interaction using the phenomenological quark models: Some studies show strong attraction which may cause a $\Omega\Omega$ bound state [15, 14], while other studies show weak repulsion [16, 17]. A recent lattice QCD analysis of the $\Omega\Omega$ scattering length [18] by using the standard finite volume method [11] shows weak repulsion in the S -wave scattering with the scattering length $a_{\Omega\Omega} = -0.16 \pm 0.22\text{fm}$: This indicates that the $\Omega\Omega$ system is unlikely to have a strongly bound dibaryon, although the large error prevents us to make a firm conclusion about details of the interaction.

The HAL QCD potential Let us first recapitulate the essential part of the HAL QCD method to be used for extracting the $\Omega\Omega$ potential. The basic quantity is the equal-time NBS wave function with the Euclidean time t ,

$$\psi_{\alpha k, \beta l}^W(\vec{r})e^{-Wt} \equiv \langle 0 | \Omega_{\alpha, k}(t, \vec{r}) \Omega_{\beta, l}(t, \vec{0}) | \Omega\Omega, W \rangle, \quad (1)$$

where $|\Omega\Omega, W\rangle$ is an exact (strangeness) = -6 state with zero total momentum. The total energy of the system is given by $W = 2\sqrt{m_\Omega^2 + \vec{p}^2}$ with the Ω baryon mass m_Ω and the relative momentum \vec{p} . Local interpolating operators for the Ω baryon, $\Omega(x)$ and $\bar{\Omega}(x)$, are taken to be

$$\begin{aligned} \Omega_{\alpha, k}(x) &\equiv \varepsilon^{abc} s_a^T(x) C \gamma_k s_b(x) s_{c, \alpha}(x), \\ \bar{\Omega}_{\alpha, k}(x) &\equiv \Omega_{\alpha, k}^\dagger \gamma^0 = -\varepsilon^{abc} \bar{s}_{a, \alpha}(x) \bar{s}_b(x) \gamma_{k1} C \bar{s}_c^T(x), \end{aligned} \quad (2)$$

where a, b and c are color indices, ε^{abc} is the totally anti-symmetric tensor, γ_k represents the Dirac matrix, and α is the spinor index. The charge conjugation matrix is taken as $C \equiv \gamma_4 \gamma_2 = -C^{-1} = -C^T = -C^\dagger$ in the Euclidean space.

An important property of the NBS wave function is its asymptotic behavior at large distance, denoted simply as

$$\psi^W(\vec{r}) \sim \sum_{L, M} e^{i\delta_L(p)} \frac{\sin(pr - \frac{L\pi}{2} + \delta_L(p))}{pr} C_{L, M} Y_{LM}(\vec{\Omega}), \quad (3)$$

where $p = |\vec{p}|$, $r = |\vec{r}|$, $\vec{\Omega}$ is the solid angle of \vec{r} , Y_{LM} is the spherical harmonic function, and L is the orbital angular momentum. Although Eq.(3) looks like a quantum mechanical formula, it can be derived from the unitarity of the S -matrix in quantum field theory with $\delta_L(p)$ being the scattering phase shift for given quantum numbers in QCD [10].

Our next task is to define the potential from which this scattering phase shift can be calculated. In the HAL QCD method, such a potential is defined through the Schrödinger type equation,

$$(E - H_0)\psi^W(\vec{r}) = \int d^3r' U(\vec{r}, \vec{r}') \psi^W(\vec{r}'), \quad (4)$$

where $H_0 \equiv -\frac{1}{2\mu_\Omega} \nabla^2$ is the free Hamiltonian, $\mu_\Omega \equiv m_\Omega/2$ is the reduced mass, and $E \equiv \frac{1}{2\mu_\Omega} p^2$ is the kinetic energy. Here $U(\vec{r}, \vec{r}')$ is the non-local but energy independent potential, which can be expanded in terms of the non-locality (the velocity or derivative expansion) [19]. The convergence of the velocity expansion at low energies has been investigated previously for the nucleon-nucleon scattering [20] and the pion-pion scattering [21]. Since we consider the low-energy S -wave scattering much below the meson production threshold in this letter, we take only the leading-order local potential, $V(\vec{r})$, in the expansion,

$$U(\vec{r}, \vec{r}') = V(\vec{r}) \delta(\vec{r} - \vec{r}') + \mathcal{O}(\vec{\nabla}).$$

Note that $V(\vec{r})$ is an effective central potential, which contains not only the genuine central part but also the tensor part implicitly [10].

The NBS wave function can be extracted from the asymptotic temporal behavior of the four-point (4-pt) function,

$$F(\vec{r}, t - t_0) = \langle 0 | \Omega(t, \vec{r}) \Omega(t, \vec{0}) \mathcal{J}(t_0) | 0 \rangle = \sum_n \langle 0 | \Omega(t, \vec{r}) \Omega(t, \vec{0}) | \Omega\Omega, W_n \rangle \langle \Omega\Omega, W_n | \mathcal{J}(t_0) | 0 \rangle + \dots$$

$$= \sum_n a_n \psi^{W_n}(\vec{r}) e^{-W_n(t-t_0)} + \dots \simeq a_0 \psi^{W_0}(\vec{r}) e^{-W_0(t-t_0)}, \quad (t - t_0 \rightarrow \infty), \quad (5)$$

where $\mathcal{J}(t_0)$ is the wall source operator which creates the $\Omega\Omega$ state at t_0 , a_n is the matrix element defined by $\langle \Omega\Omega, W_n | \mathcal{J}(0) | 0 \rangle$, W_n are discrete QCD eigen-energies in the finite volume below inelastic threshold, and W_0 is the lowest eigen-energy. The ellipses in the above equation represent inelastic contributions in the $\Omega\Omega$ system, which are suppressed for large $t - t_0$.

Eq. (5) shows that the NBS wave function for the ground state can be extracted from the 4-pt function at large t , as long as other contributions from $W_{n \geq 1} > W_0$ can be neglected. In practice, however, the increasing statistical errors of the 4-pt function at large t make it difficult to achieve the ground state saturation with reasonable accuracy [22]. Moreover, as the volume increases, larger and larger t becomes necessary to extract $\psi^{W_0}(\vec{r})$. These problems can be simultaneously avoided by the time-dependent HAL QCD method introduced in [12], where we start with the so-called R -correlator,

$$R(\vec{r}, t - t_0) \equiv \frac{F(\vec{r}, t - t_0)}{e^{-2m_\Omega(t-t_0)}} = \sum_n a_n \psi^{W_n}(\vec{r}) e^{-\Delta W_n(t-t_0)} + \dots, \quad (6)$$

with $\Delta W_n = W_n - 2m_\Omega$. Since $\Delta W_n = \frac{\vec{p}_n^2}{m_\Omega} - \frac{\Delta W_n^2}{4m_\Omega}$, we have

$$\begin{aligned} -\frac{\partial}{\partial t} R(\vec{r}, t) &\simeq \sum_n \Delta W_n a_n \psi^{W_n}(\vec{r}) e^{-\Delta W_n t} = \sum_n \left(\frac{\vec{p}_n^2}{m_\Omega} - \frac{1}{4m_\Omega} \frac{\partial^2}{\partial t^2} \right) a_n \psi^{W_n}(\vec{r}) e^{-\Delta W_n t} \\ &= \left(-\frac{1}{m_\Omega} \nabla^2 - \frac{1}{4m_\Omega} \frac{\partial^2}{\partial t^2} \right) R(\vec{r}, t) + \int d^3 r' U(\vec{r}, \vec{r}') R(\vec{r}', t), \end{aligned} \quad (7)$$

where we have replaced \vec{p}_n^2/m_Ω by $U - \nabla^2/m_\Omega$ using Eq.(4). We then arrive at the time-dependent equation,

$$\left(\frac{1}{m_\Omega} \nabla^2 - \frac{\partial}{\partial t} + \frac{1}{4m_\Omega} \frac{\partial^2}{\partial t^2} \right) R(\vec{r}, t) \simeq \int d^3 r' U(\vec{r}, \vec{r}') R(\vec{r}', t), \quad (8)$$

where "simeq" implies that we have neglected inelastic contributions by taking sufficiently large $t - t_0$. Eq.(8) gives $U(\vec{r}, \vec{r}')$ directly from $F(\vec{r}, t)$. This method no more requires the ground state saturation, so that the moderately large values of $t - t_0$ which suppress inelastic contributions are enough for a reliable extraction of the potential. Then, in the leading order of the velocity expansion, we obtain

$$V(\vec{r}) = \frac{\left(\frac{1}{m_\Omega} \nabla^2 - \frac{\partial}{\partial t} + \frac{1}{4m_\Omega} \frac{\partial^2}{\partial t^2} \right) R(\vec{r}, t)}{R(\vec{r}, t)}. \quad (9)$$

Interpolating operators for $\Omega\Omega$ system The present system can be characterized by the total spin (S), the orbital angular momentum (L), the total angular momentum (J), and the parity P . The asymptotic $\Omega\Omega$ state with given L and S has a factor $(-1)^{S+L+1}$ under the exchange of two Ω 's, so that $S + L$ must be even due to the Fermi statistics of Ω baryons. In Table 1, we show low- J channels $^{2S+1}L_J$ which appear for given conserved quantum numbers J and P . In this letter, we employ the wall source, $L = 0$ with $S = 0$ at $t = t_0$, which creates the $J^P = 0^+$ state, so that only the upper left corner of this table is considered. Then, both 1S_0 and 5D_0 channels appear after the QCD interactions at $t > t_0$. As mentioned before, we determine only the effective central potential from the 1S_0 channel at $t > t_0$, where effects of the 5D_0 state are implicitly included.

The interpolating operators for Ω with $S = 3/2$ and $S_z = \pm 3/2, \pm 1/2$ read

$$\Omega_{\frac{3}{2}, \frac{3}{2}} = -(\psi \Gamma_+ \psi) \psi_{\frac{1}{2}}, \quad (10)$$

	$P = +$	$P = -$
$J = 0$	$^1S_0, ^5D_0$	$^3P_0, ^7F_0$
$J = 1$	5D_1	$^3P_1, ^7F_1$
$J = 2$	$^5S_2, ^1D_2, ^5D_2, ^5G_2$	$^3P_2, ^7P_2, ^3F_2, ^7F_2, ^7H_2$
$J = 3$	$^5D_3, ^5G_3$	$^7P_3, ^3F_3, ^7F_3, ^7H_3$
$J = 4$	$^5D_4, ^1G_4, ^5G_4, ^5I_4$	$^7P_4, ^3F_4, ^7F_4, ^3H_4, ^7H_4, ^7J_4$

Table 1: A relation between conserved quantum numbers (J and P) and quantum numbers in the asymptotic $\Omega\Omega$ channel.

$$\Omega_{\frac{3}{2}, \frac{1}{2}} = \frac{1}{\sqrt{3}}[\sqrt{2}(\psi\Gamma_Z\psi)\psi_{\frac{1}{2}} + (\psi\Gamma_+\psi)\psi_{-\frac{1}{2}}], \quad (11)$$

$$\Omega_{\frac{3}{2}, -\frac{1}{2}} = \frac{1}{\sqrt{3}}[\sqrt{2}(\psi\Gamma_Z\psi)\psi_{-\frac{1}{2}} + (\psi\Gamma_-\psi)\psi_{\frac{1}{2}}], \quad (12)$$

$$\Omega_{\frac{3}{2}, -\frac{3}{2}} = (\psi\Gamma_-\psi)\psi_{-\frac{1}{2}}, \quad (13)$$

where $\Gamma_{\pm} \equiv \frac{1}{2}(C\gamma^2 \pm iC\gamma^1)$ and $\Gamma_Z \equiv \frac{-i}{\sqrt{2}}C\gamma^3$. We take only upper two components in the Dirac representation for the quark operators, so that the Ω operator does not have the $S = 1/2$ component. Combining these operators, the interpolating operators for the $\Omega\Omega$ system with the total spin $S = 3, 2, 1, 0$ with $S_z = 0$ are given by

$$(\Omega\Omega)_{3,0} = \frac{1}{\sqrt{20}}(\Omega_{\frac{3}{2}, \frac{3}{2}}\Omega_{\frac{3}{2}, -\frac{3}{2}} + 3\Omega_{\frac{3}{2}, \frac{1}{2}}\Omega_{\frac{3}{2}, -\frac{1}{2}} + 3\Omega_{\frac{3}{2}, -\frac{1}{2}}\Omega_{\frac{3}{2}, \frac{1}{2}} + \Omega_{\frac{3}{2}, -\frac{3}{2}}\Omega_{\frac{3}{2}, \frac{3}{2}}), \quad (14)$$

$$(\Omega\Omega)_{2,0} = \frac{1}{2}(\Omega_{\frac{3}{2}, \frac{3}{2}}\Omega_{\frac{3}{2}, -\frac{3}{2}} + \Omega_{\frac{3}{2}, \frac{1}{2}}\Omega_{\frac{3}{2}, -\frac{1}{2}} - \Omega_{\frac{3}{2}, -\frac{1}{2}}\Omega_{\frac{3}{2}, \frac{1}{2}} - \Omega_{\frac{3}{2}, -\frac{3}{2}}\Omega_{\frac{3}{2}, \frac{3}{2}}), \quad (15)$$

$$(\Omega\Omega)_{1,0} = \frac{1}{\sqrt{20}}(3\Omega_{\frac{3}{2}, \frac{3}{2}}\Omega_{\frac{3}{2}, -\frac{3}{2}} - \Omega_{\frac{3}{2}, \frac{1}{2}}\Omega_{\frac{3}{2}, -\frac{1}{2}} - \Omega_{\frac{3}{2}, -\frac{1}{2}}\Omega_{\frac{3}{2}, \frac{1}{2}} + 3\Omega_{\frac{3}{2}, -\frac{3}{2}}\Omega_{\frac{3}{2}, \frac{3}{2}}), \quad (16)$$

$$(\Omega\Omega)_{0,0} = \frac{1}{2}(\Omega_{\frac{3}{2}, \frac{3}{2}}\Omega_{\frac{3}{2}, -\frac{3}{2}} - \Omega_{\frac{3}{2}, \frac{1}{2}}\Omega_{\frac{3}{2}, -\frac{1}{2}} + \Omega_{\frac{3}{2}, -\frac{1}{2}}\Omega_{\frac{3}{2}, \frac{1}{2}} - \Omega_{\frac{3}{2}, -\frac{3}{2}}\Omega_{\frac{3}{2}, \frac{3}{2}}). \quad (17)$$

In this letter, we use Eq.(17) to calculate the $S = 0, S_z = 0$ state.

To extract the $L = 0$ state at the sink t on the lattice, we employ the cubic-group projection defined by

$$P_{\nu}^a = \frac{d_a}{g} \sum_i^g D_{\nu\nu}^a(R_i)^* R_i, \quad (18)$$

where a represents an irreducible representation of the cubic group with dimension d_a , R_i is an element of the cubic group acting on \vec{r} of Ω operators, $D^a(R_i)$ is the corresponding matrix in the irreducible representation acting on the spin of Ω operators, and g is the order of the cubic group. We use the A_1 projection, which generates the $L = 0$ as well as the $L = 4, 6, \dots$ states, where the $L = 4, 6, \dots$ components are expected to be negligibly small. For example, we have

$$P_{\nu}^{A_1}\Omega_{0,0}(\vec{r}) = \frac{1}{24} \sum_{i=1}^{24} \Omega_{0,0}(R_i\vec{r}). \quad (19)$$

The $\Omega\Omega$ potential We employ 399 gauge configurations generated by the PACS-CS Collaboration with the renormalization group improved gauge action and the non-perturbatively $\mathcal{O}(a)$ improved Wilson quark action in 2+1 flavor QCD [23]. These configurations were obtained at $\beta = 1.90$ ($a = 0.0907(13)$ fm) on the $32^3 \times 64$ lattice, whose physical extension is $L = 2.9$ fm, with the hopping parameters $\kappa_{ud} = 0.13700$

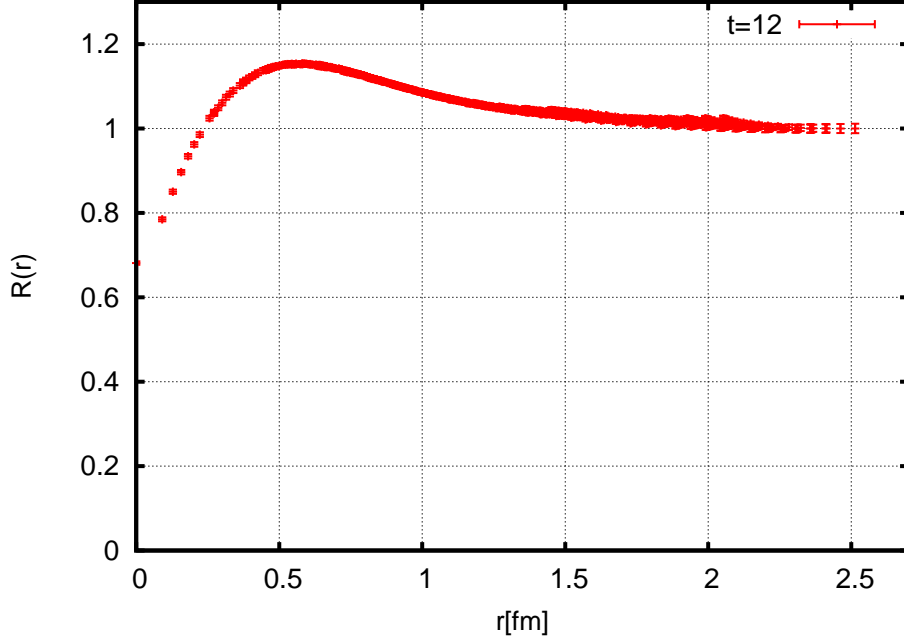


Figure 1: The R -correlator in the 1S_0 channel as a function of the relative distance r for $t - t_0 = 12$.

and $\kappa_s = 0.13640$, corresponding to $m_\pi = 701(5)$ MeV and $m_\Omega = 1966(6)$ MeV. We employ the wall quark-source with Coulomb gauge fixing. The periodic (Dirichlet) boundary condition is imposed in spacial (temporal) direction. To improve the statistics, we perform the measurement at 64 source time slices for each configuration, where the unified contraction algorithm [24] is used to calculate the NBS wave functions. Statistical errors are estimated by the Jack-Knife method. We make analyses with the bin sizes of 1, 3, 7, 19, 21 and 57, and the bin size dependence is found to be negligible. Hereafter, we show the results obtained with the bin size of 1, unless otherwise indicated.

Fig. 1 shows the R -correlator (Eq.(6)) in the 1S_0 channel as a function of r for $t - t_0 = 12$, where the R -correlator is normalized to be 1 at $r = 2.5$ fm. We find that the R -correlator is enhanced at intermediate distance, while it is suppressed at short distance. The latter behavior is consistent with the partial Pauli blocking in the quark level, similar to the situations in the nucleon-nucleon force [13].

Shown in Fig. 2 is the effective central potential $V_c(r)$ between Ω baryons at $t - t_0 = 12$ in the 1S_0 channel. The Laplacian term and the time derivative term calculated from the R -correlator in the right hand side of Eq. (8) are separately plotted in the figure, together with the total potential. We here approximate the time derivative term in Eq.(9) by $\frac{\partial}{\partial t}R(t) = \frac{R(t+1)-R(t-1)}{2}$ and $\frac{\partial^2}{\partial t^2}R(t) = R(t+1) + R(t-1) - 2R(t)$. We find that the time derivative terms have sizable contributions to the total potential: The 1st derivative in t dominates over the 2nd derivative in t . The latter corresponding to the relativistic effect is negligible. Fig. 3 shows the time dependence of $V_c(r)$ at $t - t_0 = 11, 12, 13$. This particular region of t is chosen to suppress contaminations from excited states in the single Ω propagator at smaller t and simultaneously to avoid statistical errors at larger t . We observe that the potential is nearly independent of t within statistical errors, as expected in the time-dependent method [12].

The effective central potential $V_c(r)$ in Fig. 3 has a repulsive core at short distance and an attractive well at intermediate distance. This is qualitatively similar to the NN and $\Xi\Xi$ interactions, where the partial Pauli blocking in the quark level takes place and the R -correlators are suppressed at short distance. On the other hand, no Pauli blocking is active for the H dibaryon and the $N\Omega$ dibaryon, so that there is no repulsive core in these cases [5, 9].

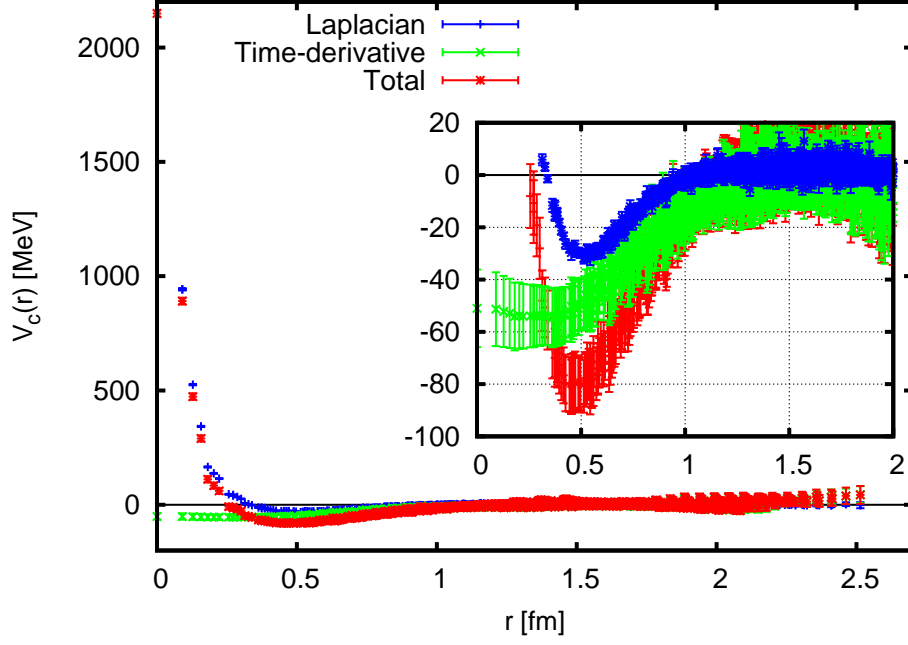


Figure 2: The effective central potential for the $\Omega\Omega$ system in the 1S_0 channel at $t-t_0 = 12$. We separately plot the Laplacian term (blue), the time derivative term (green) and the total (red).

The $\Omega\Omega$ phase shift To obtain the S -wave phase shift, we fit the potential in Fig.3 using a function which contains two Gaussian terms plus the Yukawa squared term with a form factor [13], given by

$$V_c(r) = a_1 e^{-a_2 r^2} + a_3 e^{-a_4 r^2} + a_5 (1 - e^{-a_6 r^2})^2 \left(\frac{e^{-a_7 r}}{r} \right)^2, \quad \lim_{r \rightarrow 0} V(r) = a_1 + a_3. \quad (20)$$

This 2 Gauss + (Yukawa)² form gives a fairly good fit with $\chi^2/\text{d.o.f} \sim 0.50$ at $t - t_0 = 12$. The resulting parameters are $a_1 = 1.69(6) \times 10^3 \text{ MeV}$, $a_2 = 1.24(3) \times 10^2 \text{ fm}^{-2}$, $a_3 = 4.44(68) \times 10^2 \text{ MeV}$, $a_4 = 5.68(1.31) \text{ fm}^{-2}$, $a_5 = -7.06(14.64) \times 10^4 \text{ MeV}$, $a_6 = 6.25(5.77) \times 10^{-1} \text{ fm}^{-2}$, $a_7 = 3.43(30) \text{ fm}^{-1}$ at $t - t_0 = 12$.

Using the fitted potential, we solve the Schrödinger equation in the infinite volume to calculate the scattering phase shift $\delta_L(k)$ of the $\Omega\Omega$ system in 1S_0 channel by the formula with $L = 0$,

$$\tan \delta_L(k) = \lim_{x_1, x_2 \rightarrow \infty} \frac{\psi_k(x_2) \sin(kx_1 - \frac{L\pi}{2}) - \psi_k(x_1) \sin(kx_2 - \frac{L\pi}{2})}{\psi_k(x_1) \cos(kx_2 - \frac{L\pi}{2}) - \psi_k(x_2) \cos(kx_1 - \frac{L\pi}{2})}, \quad (21)$$

where ψ_k is the wave function and k is the magnitude of the momentum.

Fig. 4 shows the phase shift as a function of the kinetic energy, $E = k^2/m_\Omega$. The result indicates that the $\Omega\Omega$ interaction is attractive at low energies, while the existence of the bound state is inconclusive because of large statistical errors near $k = 0$. Indeed, the central value of $\delta_0(k = 0)$ at $t - t_0 = 11$ is 180° , while it becomes zero at $t - t_0 = 12, 13$. Due to the same reason, the scattering length and the effective range cannot be extracted reliably from this phase shift. A possible physical interpretation of such a situation is that the $\Omega\Omega$ system at the present quark masses may appear close to the unitary limit where the scattering length diverges and changes its sign.

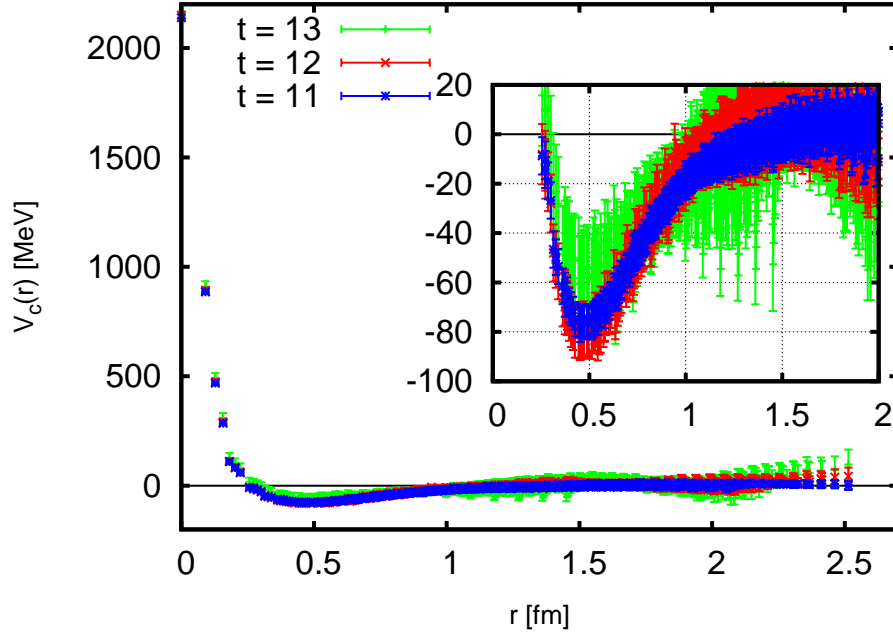


Figure 3: The effective central potential $V_c(r)$ in the 1S_0 channel at $t-t_0 = 11$ (blue), 12 (red), 13 (green).

Conclusion In this letter, we have calculated the effective central potential and the scattering phase shift between Ω baryons in the 1S_0 channel by using the HAL QCD method. We have found that the $\Omega\Omega$ potential has short range repulsion and intermediate range attraction just like the nucleon-nucleon potential. The short range repulsion of this system is a reflection of the partial Pauli blocking in the quark level similar to the nucleon-nucleon potential, and is in contrast to the cases of the H -dibaryon or $N\Omega$ system with no repulsion. The $\Omega\Omega$ interaction is attractive at low energies, but is not strong enough to form a tightly bound dibaryon at quark masses corresponding to $m_\pi \simeq 700$ MeV and $m_\Omega \simeq 1970$ MeV. Rather, the system may appear close to the unitary limit at these quark masses. We plan to carry out the 2+1 flavor simulations at the physical quark masses, in order to investigate whether the attraction found in the present study increases or decreases toward the physical quark masses.

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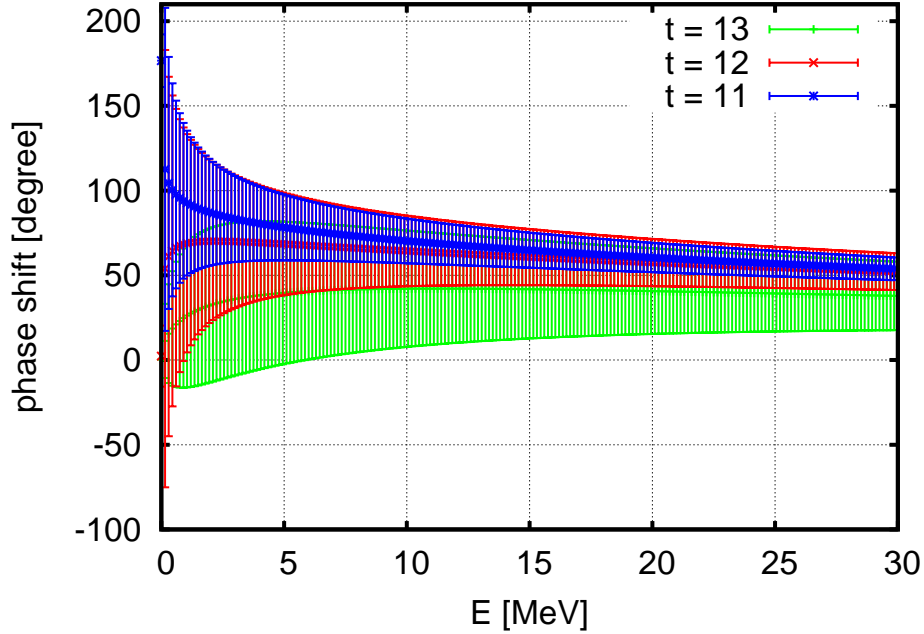


Figure 4: Phase shift $\delta_0(k)$ of the $\Omega\Omega$ in the (1S_0) channel at $t - t_0 = 11$ (blue), 12 (red), 13 (green).

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